Problem 1.25

Show that every function $f(t)$ can be represented as a sum of an odd function $O(t)$ and an even function $E(t)$.

Solution

$$\begin{aligned} f(t) &= \frac{f(t)}{2} + \frac{f(t)}{2} \\ &= \frac{f(t) - f(-t)}{2} + \frac{f(t) + f(-t)}{2} \end{aligned}$$

The first fraction is an odd function:

$$\begin{aligned} O(t) = \frac{f(t) - f(-t)}{2} \quad \rightarrow \quad O(-t) &= \frac{f(-t) - f(t)}{2} \\ &= -\frac{f(t) - f(-t)}{2} \\ &= -O(t). \end{aligned}$$

The second fraction is an even function:

$$\begin{aligned} E(t) = \frac{f(t) + f(-t)}{2} \quad \rightarrow \quad E(-t) &= \frac{f(-t) + f(t)}{2} \\ &= \frac{f(t) + f(-t)}{2} \\ &= E(t). \end{aligned}$$

Therefore, every function can be represented as a sum of an odd function and an even function.

$$f(t) = O(t) + E(t)$$